Hessel Posthuma
Korteweg-de Vries Institute for Mathematics, University of Amsterdam, H.B.Posthuma(at)uva.nl

Noncommutative models for singular spaces

Abstract:

I will give an introduction to the noncommutative geometry approach to singular spaces. I will start by giving an introduction to the basic tools of noncommutative geometry, such as K-theory and Hochschild and cyclic cohomology. The main examples of singular spaces that I will discuss come from orbifolds and foliations, for which I will introduce the language of groupoids. This is essential for the noncommutative geometry description of such spaces, which starts from the convolution algebra of the corresponding groupoid. With this description, we can discuss quantization and index theory.

Johannes Huebschmann
Université des Sciences et Technologies de Lille, Johannes.Huebschmann(at)math.univ-lille1.fr

Poisson-Kähler geometry of stratified Kähler spaces and quantization

Abstract:

In the presence of classical phase space singularities the standard methods are insuficient to attack the problem of quantization. In certain situations these difficulties can be overcome by means of stratified Kähler spaces. Such a space is a stratified symplectic space together with a complex analytic structure which is compatible with the stratified symplectic structure; in particular each stratum is a Kähler manifold in an obvious fashion.
Examples abound: Symplectic reduction, applied to Kähler manifolds, yields a particular class of examples; this includes adjoint and generalized adjoint quotients of complex semisimple Lie groups which, in turn, underly certain lattice gauge theories. Other examples come from certain moduli spaces of holomorphic vector bundles on a Riemann surface and variants thereof; in physics language, these are spaces of conformal blocks. Still other examples arise from the closure of a holomorphic nilpotent orbit. Symplectic reduction carries a Kähler manifold to a stratified Kähler space in such a way that the sheaf of germs of polarized functions coincides with the ordinary sheaf of germs of holomorphic functions. Projectivization of holomorphic nilpotent orbits yields exotic stratified Kähler structures on complex projective spaces and on certain complex projective varieties including complex projective quadrics. Other physical examples are reduced spaces arising from angular momentum, including our solar system whose correct reduced phase space acquires the structure of an affine stratified Kähler space. In the presence of singularities, the naive restriction of the quantization problem to a smooth open dense part, the \top stratum", may lead to a loss of information and in fact to inconsistent results. Within the framework of holomorphic quantization, a suitable quantization procedure on stratified Kähler spaces unveils a certain quantum structure having the classical singularities as its shadow. The new structure which thus emerges is that of a costratified Hilbert space, that is, a Hilbert space together with a system which consists of the subspaces associated with the strata of the reduced phase space and of the corresponding orthoprojectors. The costratified Hilbert space structure re ects the stratification of the reduced phase space. Given a Kähler manifold, reduction after quantization then coincides with quantization after reduction in the sense that not only the reduced and unreduced quantum phase spaces correspond but the invariant unreduced and reduced quantum observables as well. We will illustrate the approach with a concrete model: We will present a quantum (lattice) gauge theory which incorporates certain classical singularities. The reduced phase space is a stratified Kähler space, and we make explicit the requisite singular holomorphic quantization procedure and spell out the resulting costratified Hilbert space. In particular, certain tunneling probabilities between the strata emerge, the energy eigenstates can be determined, and corresponding expectation values of the orthoprojectors onto the subspaces associated with the strata in the strong and weak coupling approximations can be explored.

References
[1] J. Huebschmann: Kähler spaces, nilpotent orbits, and singular reduction. Memoirs of the AMS 172 (814), Amer. Math. Soc., Providence R.I., , 2004, math.DG/0104213
[2] J. Huebschmann: Kähler quantization and reduction. J. reine angew. Math. 591, 75{109 (2006), math.SG/0207166
[3] J. Huebschmann, G. Rudolph and M. Schmidt: A gauge model for quantum mechanics on a stratified space. Commun. Math. Physics 286 (2009), 459{494, hep-th/0702017

Eugene Lerman

University of Illinois at Urbana-Champaign, lerman(at)math.uiuc.edu

Differential characters and prequantization

Abstract:

Prequantization of a symplectic manifold M with an integral symplectic form w is often informally described as assigning to (M,w) a principal circle bundle P over M together with a connection A whose curvature is w. But this is not exactly true. For instance, there may be more than one integral cohomology class c whose image in the de Rham cohomology is the class of w.

Furthermore, if (M,w) carries an action of a discrete group G, it is far from clear that this action to a connection preserving action of G on P. Intrinsic prequantization of orbifolds is even more mysterious. An elegant solution to these problems can be found in a paper of Hopkins and Singer. Regard the collection of all principal circle bundles with connections over M as a category. Extend the collection of integral closed 2-forms on M to a category as well by twisting them with integral cocycles (this is a categorification of Cheeger-Simons differential characters) and turn prequantization into a functor Preq_M from characters to bundles. This functor is natural in the manifold M and consequently can be extended from manifolds to Lie groupoids, hence to orbifolds.