MASTERCLASS 18-22 MARCH 2013

BY **Ben Elias**, MIT, Boston and **Geordie Williamson**, MPI, Bonn

Titel: Soergel bimodules & Kazhdan-Lusztig conjectures

Program

Venue: aud. G1

Monday 18 March, "The Cast"

9.30-10.00	Coffee/tea

10.00-10.45   **Introduction: Category *O* and the Kazhdan-Lusztig conjectures**

The happenings of 1979. The miracle of KL polynomials. Arbitrary Coxeter groups. The miracle of the localisation proof. Soergel's dream of an algebraic explanation... the deepening mystery of positivity.

11.15-12.00   **Hecke algebras and Kazhdan-Lusztig polynomials**

The Coxeter complex. The Hecke algebra of a Coxeter group. The presentation using standard generators. The standard basis. The Kazhdan-Lusztig basis and polynomials. Products of Kazhdan-Lusztig generators and the defect formula.

12.00-14.00   Lunch/exercises in Kol-G 1532-214

14.00-14.45   **Soergel bimodules**

Invariant theory for finite reflection groups. Bimodules and monoidal categories. The category of Soergel bimodules. First examples.

14.45-15.15   Afternoon tea

15.15-16.00   **How to draw monoidal categories**

Higher algebra. Drawing adjunctions, cyclicity etc. Example: 2-groupoids. The Coxeter groupoid. The Zamolodchikov relations.

16.00 -   Exercises in Kol-G 1532-214

18.00-   Light supper and wine (in MatLab, follow the signs)

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Tuesday 19 March, "Getting to know Soergel bimodules"

9.30-10.00	Coffee/tea
10.00-10.45	**The classical approach to Soergel bimodules**
	Standard bimodules. Support filtrations. Soergel's hom formula. Statement of Soergel's categorification theorem. Discussion.
11.15-12.00	**The dihedral cathedral**
	Starting to draw Soergel bimodules. Soergel bimodules in rank 2. Jones-Wenzl projectors, connections to the Temperley-Lieb algebra and quantum groups. Categorification of the Kazhdan-Lusztig presentation.
12.00-14.00	Lunch/exercises in Kol-G 1532-214
14.00-14.45	**Generators and relations, the light leaves basis**
	Generators and relations in general. Light leaves morphisms as a categorification of the defect formula. Double leaves give a basis for morphisms.
14.45-15.15	Afternoon tea
15.15-16.00	**How to draw Bott-Samelson bimodules**
	Zero-one sequences and multiplication. Intersection forms. Duality. Discussion.
16.00	Exercises in Kol-G 1532-214

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Wednesday 20 March, "Soergel bimodules and glimpses of geometry"

9.30-10.00	Coffee/tea
10.00-10.45	**Soergel's categorification theorem**

The cellular structure. A discussion of idempotent lifting. Generators and relations proof of Soergel's categorification theorem. Examples of intersection forms and idempotents.

11.15-12.00   **Hodge theory and Lefschetz linear algebra**

Review of the (real) Hodge theory of smooth projective algebraic varieties. A discussion of the weak and hard Lefschetz theorems. Lefschetz operators, Lefschetz forms and the Hodge-Riemann bilinear relations. Tricks establishing the Lefschetz package. The weak Lefschetz substitute.

12.00-14.00   Lunch/exercises in Kol-G 1532-214

14.00-14.45   **The Hodge theory of Soergel bimodules**

Statement of the results and outline of the methods. The embedding theorem, the limit argument. The absence of the weak Lefschetz theorem.

14.45-15.15   Afternoon tea

15.15-16.00   **Lightning introduction to IC, H and Soergel bimodules**

Varieties stratified by affine spaces and the constructible derived category. How to compute stalks of a proper push-forward. Poincaré duality. Stalks definition of an IC sheaf. The connection to Kazhdan-Lusztig polynomials on the flag variety. Global sections and Soergel bimodules.

16.00 -   Exercises in Kol-G 1532-214

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Thursday 21 March, "Soergel's Conjecture and the Kazhdan-Lusztig conjecture"

9.30-10.00	Coffee/tea
10.00-10.45	**Rouquier complexes and homological algebra**
	The homotopy category of Soergel bimodules. Minimal complexes. Rouquier complexes. Examples.
11.15-12.00	**Proof of hard Lefschetz**
	The perverse filtration on Soergel bimodules. The diagonal miracle. Factoring the Lefschetz operator. Hard Lefschetz.
12.00-14.00	Lunch/exercises in Kol-G 1532-214
14.00-14.45	**Lightning introduction to category O and Soergel's V.**
	Review of Verma modules, category O and its block decomposition by central character. Statement of the Kazhdan-Lusztig conjecture. Soergel's functor V. Soergel's conjecture implies the Kazhdan-Lusztig conjecture.
14.45-15.15	Afternoon tea
15.15-16.00	**Overflow/discussion session**
16.00 -	Exercises in Kol-G 1532-214
18.00-22.30	Special dinner at a restaurant in Aarhus City: "Ministeriet"

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Friday 22 March, Discussion and Applications

9.30-10.00	Coffee/tea
10.00-10.45	*Suggestions from the participants are welcome!*
11.15-12.00	*Suggestions from the participants are welcome!*
12.00-14.00	Lunch/exercises in Kol-G 1532-214
14.00-14.45	*Suggestions from the participants are welcome!*
14.45-15.15	Afternoon tea
15.15-16.00	**Overflow/ discussion session**
16.00	Time for a beer ☺

Some ideas for Friday lectures:

Hecke algebras with unequal parameters and folding. Definition of Hecke algebras with unequal parameters. Equivariant K-theory. Categorification of unequal parameters in the quasi-split case.

The situation in characteristic p. Lusztig's conjecture. Intersection forms. The p-canonical basis. Examples. Many mysteries and open questions.

de Cataldo and Migliorini's work in the non semi-small case. Two Lefschetz operators. The perverse filtration. The relative hard Lefschetz theorem. EW conjectures which will hopefully be theorems soon!

Categorifications of braid groups. Categorifying the braid group. Generators and relations for strict braid group actions. Deligne's theorem and the EW version. Topological explanation for Zamolodchikov relations.

Algebraic quantum geometric Satake. A discussion of ridiculous titles. Algebraizing the geometric Satake equivalence. Quantizing it in type A using Ben's favorite Cartan matrix.