REFERENCES FOR
QUANTIZATION OF MODULI SPACES
GEOQUANT SUMMER SCHOOL AT QGM, 2017.

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ABSTRACT. This list of references are compiled for the students of the *Quantization of Moduli Spaces*-class of the GeoQuant summer school at QGM 2017. It is by no means an extensive list of reference on the subject covered by the class, but just a suggestion for a starting point with most relevance for this class.

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